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Leader-Follower Consensus with Unknown Control Direction

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***Abstract*—The paper considers leader-follower consensus of multi-agent networks with unknown control direction. Sliding mode control is used to achieve consensus tracking under fixed topology with the assumption that the position of the leader is known to a subset of the followers. The proposed consensus law assumes unknown sign in the control input matrix of the followers and does not require the knowledge of the leader's velocity. Lyapunov-based analysis is presented to show that if the directed graph of the network has a directed spanning tree then sliding mode control law can guarantee consensus tracking. Simulations results are provided to verify the feasibility of the proposed controller.**

I. INTRODUCTION

Multi-agent systems excel in cooperatively accomplishing tasks that may not be executable using any single agent [1], which makes these systems attractive to a variety of military and civilian applications, e.g., [2]–[5]. Design and analysis of cooperative strategies for multi-agent systems can be challenging due to uncertainties in the environment (e.g., exogenous disturbances), unmodeled dynamics, or parametric uncertainties. If multiple agents, such as quadrotors or ground vehicles, are defectively produced in batches, then it is not uncommon to assume that they have model uncertainty as well as unknown control direction [6]. To achieve stable cooperative behavior, it is crucial to design controllers that compensate for modeling, environmental as well as manufacturing anomalies.

Sliding mode control has been successfully used in cooperative multi-agent systems to compensate for model uncertainties and exogenous disturbances. Fuzzy sliding mode controllers [7], [8], robust consensus controllers [9], [10], and robust leaderless consensus controllers [11], [12] have been recently developed for various leader-follower systems. Although successful in their research endeavors, the existing solutions using sliding mode control in cooperative multi-agent systems do not take into account sign uncertainties in the input matrix.

The Nussbaum-type function has been extensively used in systems with unknown control direction [13]–[15]. Stability analysis for a single system using Nussbaum-type function is relatively straightforward. However, for cooperative systems, where each system contains an unknown control direction, the analysis can be challenging (see [6], [15], [16]). Additionally, the adaptive control approaches using Nussbaum-type function in [6], [15], [16] can only guarantee asymptotic consensus.

The contribution of this paper is in the development of an exponentially stable robust consensus controller for a second order system and a finite time stable robust controller for the first order systems in the presence of unknown control direction and non-vanishing disturbances. Leveraging the efforts in [17], for leader-follower multi-agent systems, consensus is guaranteed through periodic switching laws even when the followers have unknown and possibly time-varying sign in their input matrix. As opposed to [18]–[20], where the velocity of the leader is assumed to be known, the proposed controller requires only an upperbound on the leader's velocity. The leader-follower topology is modeled as a weighted directed graph such that the directed spanning tree of the graph is rooted at the leader node. The Lyapunov-based analysis guarantees exponential consensus tracking of the agents. Simulations results are provided to demonstrate the efficacy of the proposed controller.

II. UNKNOWN CONTROL DIRECTION BACKGROUND

The following section provides mathematical background for the rest of the paper. An in-depth controller development can be found in [17]. The method proposed in [17], [21], [22] is based on partitioning hypersurfaces into cell-like structures. The hypersurfaces are the cellular structure of the sliding manifolds implemented through periodic switching. These manifolds are parallel switching surfaces partitioned onto the state space as cells

$$\mathbb{M}_i = \{x \in \mathbb{R}^n : \tilde{s}_i(x) = \Psi(s_i(x)) = 0\} \tag{1}$$

where $\tilde{s}_i(x) = 0$ is the traditional switching surface, and $\Psi(\cdot)$ is a switching function.

Consider a system with scalar control input as

$$\dot{x} = f(t, x) + b(t, x)\, u_s \tag{2}$$

where $x(t) \in \mathbb{R}^n$, $b(t, x) \in \mathbb{R}^n$, $u_s(t) \in \mathbb{R}$. The objective is to stabilize the system to the desired manifold $s(x) = 0$, where $s(x) \in \mathbb{R}^1$. The challenge of this system is the vector $b(t, x) = [b_1(t, x), \ldots, b_n(t, x)]^T$ defining actuation direction is time- and state-dependent, and its sign is unknown. Taking time derivative of the surface $s(x)$ yields

$$\dot{s} = G(x)f(t, x) + G(x)b(t, x)u_s$$

where $G(x) = \frac{\partial s(x)}{\partial x}$. If the vector $b(t, x)$ is known, then $u_s(t)$ can be designed so that $\dot{s}(x) < 0$, then sliding mode can occur to guarantee that $s(x)$ goes to zero. For an in-depth discussion on sliding mode control, readers are referred to [23]. To reach the sliding manifold $s(x)$ without knowledge of the input vector $b(t, x)$, the proposed control law contains a periodic switching function as

$$u_s = M_0 sign\left[\sin\left(\frac{\pi}{\varepsilon}\tilde{s}\right)\right] \tag{3}$$

where $\tilde{s}(t)$ is defined as

$$\tilde{s} = s(t) + \lambda \int_0^t sign(s(\tau))d\tau \tag{4}$$

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and $M_0 > 0$ is a positive control gain that can be a constant or a function, $\lambda \in \mathbb{R}$ is a positive constant that determines the rate of convergence of the manifolds, and $\varepsilon \in \mathbb{R}$ is a positive constant that determines the spacing between the manifolds.

If for all $t > 0$ and $x(t) \in \mathbb{R}^n$ the function $b(t,x)$ satisfies

$$G(x)b(t,x) \neq 0 \tag{5}$$

then the control in (3) can reach the surface $s(x) = 0$ in finite time, provided the variable M_0 satisfies the inequality

$$|G(x)b(t,x)M_0(t,x)| > |G(x)f(t,x)| + \lambda + c \tag{6}$$

where $c > 0$ is some positive constant. From (6), it is obvious that by increasing M_0, the control law can compensate for a larger range of uncertainty. Differentiating $\tilde{s}(t)$ in (4) and substituting the controller in (3) yields

$$\dot{\tilde{s}} = Gf + Gb^T M_0 sign\left[\sin\left(\frac{\pi}{\varepsilon}\tilde{s}\right)\right] + \lambda sign(s). \tag{7}$$

In the neighborhoods of the points where

$$\tilde{s} = k\varepsilon \tag{8}$$

for $k = 0, \pm 2, \pm 4, \ldots$, the following is obtained:

$$sign\left[\sin\left(\frac{\pi}{\varepsilon}\tilde{s}\right)\right] = sign(\tilde{s} - k\varepsilon)$$

and for $k = \pm 1, \pm 3, \ldots$, the following is obtained:

$$sign\left[\sin\left(\frac{\pi}{\varepsilon}\tilde{s}\right)\right] = -sign(\tilde{s} - k\varepsilon).$$

If the inequality in (6) is satisfied then sliding mode will occur on one of the manifolds in (8) for any sign of $G(x)b(t,x)M_0$. In fact, sliding mode occurs where $\tilde{s} = constant$ after some moment of time, and after differentiating (4) yields

$$\dot{s} = -\lambda sign(s). \tag{9}$$

Thus, (9) guarantees that the manifold $s(x) = 0$ is reached in finite time.

From the geometric point of view, there are an infinite number of parallel switching surfaces partitioned into cells (see Fig. 1) that have stable sliding manifolds for certain sign of $G(x)b(t,x)$. Thus, based on (9), all the parallel manifolds move in the direction of the manifold $s = 0$ and sliding mode is reached in finite time.

III. Leader-Follower Topology

For notation convenience, let the node set $\mathcal{V} = \{1, \ldots, n\}$ be the group of agents. The set of leaders and followers are denoted as $\mathcal{V}_L$ and $\mathcal{V}_F$, respectively, such that $\mathcal{V}_L \cup \mathcal{V}_F = \mathcal{V}$ and $\mathcal{V}_L \cap \mathcal{V}_F = \emptyset$. Let $\mathcal{V}_L = \{1\}$ and $\mathcal{V}_F = \{2, \ldots, n\}$.

Assumption 1. *The graph $\mathcal{G}$ has a directed spanning tree rooted at the leader's node.*

Assumption 2. *At least one of the followers is connected to the leader.*

The interaction among the agents are modeled as a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and $\mathcal{E}$ are the edges. The followers are modeled as a directed graph $\mathcal{G}_F = (\mathcal{V}_F, \mathcal{E}_F)$. A direct edge $(i,j) \in \mathcal{E}$ in $\mathcal{G}$ exists between node i and j if they are connected.

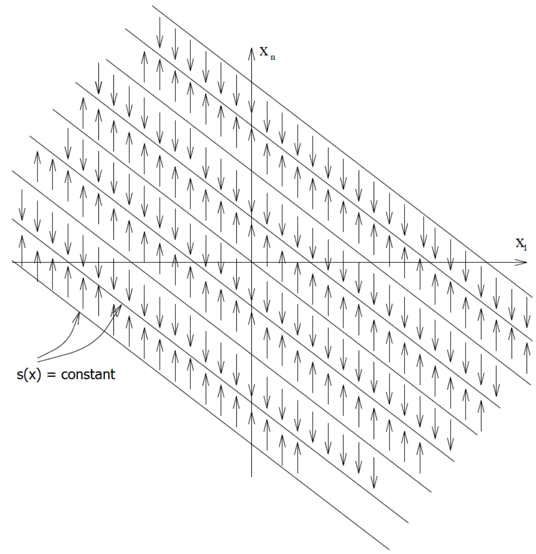

Fig. 1: Multiple Equilibrium Manifolds

The direct edge (i,j) indicates that node i can access the states of node j through local sensing, but not vice versa. Therefore, node j is a neighbor of node i. A directed spanning tree is a directed graph, where every node has one parent except for the root node. The root node has directed paths to every other node in the graph. Since the followers are not aware of the leader's intentions (e.g., its desired trajectory), they have to stay connected with the leader directly or indirectly through concatenated paths, such that the knowledge of the leader's state can be delivered to all the nodes through the connected network.

The weighted adjacency matrix $A \in \mathbb{R}^{n\times n}$ of $\mathcal{G}$ contains nonnegative elements, where $a_{ij} \geq 0$ if there is an edge between the i^{th} and j^{th} agent, and a_{ij} denotes the elements in A. Let $D \triangleq \text{diag}\{d_1, \cdots, d_n\} \in \mathbb{R}^{n\times n}$ be the diagonal matrix, where $d_i \triangleq \sum_{j=1}^{n} a_{ij}$ for $i = 1, \cdots, n$. The Laplacian of the weighted graph $\mathcal{G}$, denoted by $L \in \mathbb{R}^{n\times n}$, is defined as

$$L \triangleq D - A.$$

Let the adjacency matrix describing the topology between the leader and followers be expressed as

$$A = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \in \mathbb{R}^{n\times n} \tag{10}$$

and let the subgraph of $\mathcal{G}$ be denoted as $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$, where the sub-adjacency matrix is defined as

$$\bar{A} \triangleq \begin{bmatrix} a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots \\ a_{n2} & \cdots & a_{nn} \end{bmatrix} \in \mathbb{R}^{(n-1)\times(n-1)}. \tag{11}$$

Consider a diagonal matrix $\bar{D} = \text{diag}\{\bar{d}_2, \cdots, \bar{d}_n\}$, where $\bar{d}_i = \sum_{j=2}^{n} a_{ij}$ for $i = 2, \cdots, n$. The elements of an adjacency matrix $\bar{A}$ are represented by

$$a_{ij} = \begin{cases} 1 & \forall (i,j) \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}. \tag{12}$$

Let b_i denote the interconnection between the i^{th} agent and the leader. The connection weight among the leaders and agents can be represented by a diagonal matrix $\bar{B} = \text{diag}\{b_2, \cdots, b_n\} \in \mathbb{R}^{n-1 \times n-1}$, where

$$b_i = \begin{cases} 1 & \text{if agent } i \text{ is connected to the leader} \\ 0 & \text{otherwise} \end{cases}. \quad (13)$$

IV. Controller Development

The leader and the followers are modeled as a single integrator system subjected to disturbance as

$$\dot{x}_i = g_i u_i + \delta_i \quad \forall i = 1, \cdots, n \quad (14)$$

where $x_i(t) \in \mathbb{R}$ is the state of an i^{th} agent, $g_i(t) \in \mathbb{R}$ denotes the input gain of known magnitude but unknown sign, $u_i(t) \in \mathbb{R}$ is the control input, and $\delta_i(t) \in \mathbb{R}$ is the unknown non-vanishing disturbance. The disturbance is assumed to be bounded in the sense that

$$\|\delta_i\| \leq \bar{\Delta}_i < \infty \quad (15)$$

where $\bar{\Delta}_i \in \mathbb{R}$ is a known positive constant. Also, let $\delta = [\delta_2, \ldots, \delta_n]^T \in \mathbb{R}^{n-1 \times n-1}$, and $\bar{\Delta} = [\bar{\Delta}_2, \ldots, \bar{\Delta}_n]^T \in \mathbb{R}^{n-1 \times n-1}$.

Assumption 3. *The control input u_1 of the leader is bounded such that $|\dot{x}_1| \leq \bar{\dot{x}}_1$, where $\bar{\dot{x}}_1 \in \mathbb{R}$ is a known positive constant.*

Let $e_i(t) \in \mathbb{R}$ for $i = 2, \cdots, n$ denote the error as

$$e_i = \sum_{j=2}^{n} a_{ij}(x_i - x_j) + b_i(x_i - x_1) \quad (16)$$

where a_{ij} and b_i are defined in (12) and (13), respectively. The error in (16) can be rewritten as

$$e_i = x_i \sum_{j=2}^{n} a_{ij} - \sum_{j=2}^{n} a_{ij} x_j + b_i x_i - b_i x_1 \quad (17)$$

Consider $e = [e_2, \cdots, e_n]^T \in \mathbb{R}^{n-1}$ to be the vector of error functions. Taking time derivative of $e(t)$ along (14), equation (17) can be expressed as

$$\dot{e} = \left(\bar{D} + \bar{B}\right)\dot{x} - \bar{A}\dot{x} - \bar{B}\mathbf{1}\dot{x}_1 \quad (18)$$

where $x = [x_2, \cdots, x_n]^T \in \mathbb{R}^{n-1}$, and $\mathbf{1} = [1, \cdots, 1]^T \in \mathbb{R}^{n-1}$. For the graph $\bar{\mathcal{G}}$, using the graph Laplacian given by

$$\bar{L} = \bar{D} - \bar{A} \quad (19)$$

the open-loop error system in (18) can be expressed as

$$\dot{e} = \left(\bar{L} + \bar{B}\right)\dot{x} - \bar{B}\mathbf{1}\dot{x}_1. \quad (20)$$

Based on [24], the matrix $\bar{L} + \bar{B}$ in (20) is invertible.

Consider an augmented error vector, denoted by $\tilde{e}(t) \in \mathbb{R}^{n-1}$ as

$$\tilde{e} \triangleq e + \lambda \int_0^t sign(e(\tau))d\tau \quad (21)$$

where $\lambda \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix. When time derivative of the augmented error vector $\tilde{e}$ along (14) is zero, i.e $\dot{\tilde{e}}(t) = 0$, this implies that

$$\dot{e} = -\lambda sign(e) \quad (22)$$

and the error $e(t)$ approaches zero in finite time.

Theorem 1. *For n-agents connected with a directed graph that has a directed spanning tree, there exists a controller for the leader-follower system in (14) such that consensus tracking can be achieved in finite time even if the sign of the control direction, i.e., $sign(g_i)$, is unknown.*

Proof. Define a Lyapunov function candidate $V(t) \in \mathbb{R}$ as

$$V = \frac{1}{2}\tilde{e}^T\tilde{e} \quad (23)$$

Case 1: Consider the case when the sign of g_i is unknown but identical, and the magnitude of g_i is identical for all the agents, i.e., $g_i = g_j \triangleq g \; \forall i, j \neq 1$.

Taking time derivative of $V(t)$ and using (14), the Lyapunov derivative can be obtained as

$$\begin{aligned} \dot{V} = \tilde{e}^T\Big(&\left(\bar{L} + \bar{B}\right)gU - \bar{B}\mathbf{1}\dot{x}_1 \\ &+ \lambda sign\left(e\right) + \left(\bar{L} + \bar{B}\right)\delta\Big) \end{aligned} \quad (24)$$

where $U = [u_2, \cdots, u_n]^T \in \mathbb{R}^{n-1}$ is the control vector, and $\delta \triangleq [\delta_2, \cdots, \delta_n]^T \in \mathbb{R}^{n-1}$. For the open-loop error system in (20), the control input can be designed as

$$U = \left(\bar{L} + \bar{B}\right)^{-1}\frac{M_a}{|g|} sign\left(\sin\left(\frac{\pi}{\epsilon}\tilde{e}\right)\right) \quad (25)$$

where $M_a = diag\{M_{a2}, \cdots, M_{an}\} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix, and $\epsilon = diag\{\epsilon_2, \cdots, \epsilon_n\} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix. Using the fact that $\bar{L} + \bar{B}$ has full rank, the matrix M_a is chosen such that each element of $M_a\mathbf{1}$ is greater than the corresponding element in $|\bar{B}\mathbf{1}\bar{\dot{x}}_1| + |\lambda\mathbf{1}| + |(\bar{L} + \bar{B})\bar{\Delta}|$. Using the analysis presented in Section II, if $sign(U) = -sign(g(\bar{L}+\bar{B})e)$, then $\dot{V}$ is negative definite. From (24) and (25), for a constant vector $\beta \in \mathbb{R}^{n-1}$, the surface $\tilde{e}(t)$ reaches β in finite time and sliding mode occurs on β. Then, it follows from (22) that $e(t)$ goes to zero in finite time. Thus, consensus tracking is guaranteed to be achieved in finite time.

Case 2: The control input gains are not identical and could be time-varying, i.e. $g_i(t) \neq g_j(t) \; \forall i, j \neq 1$ and $i \neq j$.

Taking time derivative of Lyapunov function in (23),

$$\begin{aligned} \dot{V} = \tilde{e}^T\Big(&\left(\bar{L} + \bar{B}\right)GU - \bar{B}\mathbf{1}\dot{x}_1 \\ &+ \lambda sign\left(e\right) + \left(\bar{L} + \bar{B}\right)\delta\Big) \end{aligned} \quad (26)$$

where $G = \text{diag}\{g_2, \cdots, g_n\} \in \mathbb{R}^{(n-1)\times(n-1)}$. The control input can be designed as

$$U = |G|^{-1}\left(\bar{L} + \bar{B}\right)^{-1} M_b sign\left(\sin\left(\frac{\pi}{\epsilon}\tilde{e}\right)\right) \quad (27)$$

where $M_b = \text{diag}\{M_{b2}, \cdots, M_{bn}\} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix chosen such that each element of $M_b\mathbf{1}$ is greater than the corresponding element in $|\bar{B}\mathbf{1}\bar{\dot{x}}_1| + |\lambda\mathbf{1}| + |(\bar{L} + \bar{B})\bar{\Delta}|$.

The controllers in (25) and (27) guarantee finite time consensus tracking among agents with single integrator dynamics. $\square$

Extension to Second Order System

Consider the agents modeled as a double integrator system subjected to disturbance as

$$\begin{aligned} \dot{x}_{1i} &= x_{2i} \\ \dot{x}_{2i} &= g_i u_i + \delta_i \end{aligned} \tag{28}$$

where $g_i(t) \in \mathbb{R}$ and $\delta_i(t) \in \mathbb{R}$ are defined in (14). Let the tracking error for the i^{th} agent be defined as

$$e_{1i} \triangleq \sum_{j=2}^{n} a_{ij}(x_{1i} - x_{1j}) + b_i(x_{1i} - x_{11}) \tag{29}$$

$$e_{2i} \triangleq \sum_{j=2}^{n} a_{ij}(x_{2i} - x_{2j}) + b_i(x_{2i} - x_{21}) \tag{30}$$

where $\dot{e}_{1i}(t) = e_{2i}(t)$. The error system in (29) and (30) can be expressed in a concised form as

$$e_1 = \left(\bar{L} + \bar{B}\right) x_1 - \bar{B}\mathbf{1}x_{11} \tag{31}$$

$$e_2 = \left(\bar{L} + \bar{B}\right) x_2 - \bar{B}\mathbf{1}x_{21} \tag{32}$$

where $e_1 = [e_{12}, \cdots, e_{1n}]^T \in \mathbb{R}^{n-1}$, $e_2 = [e_{22}, \cdots, e_{2n}]^T \in \mathbb{R}^{n-1}$, $x_1 = [x_{12}, \cdots, x_{1n}]^T \in \mathbb{R}^{n-1}$, and $x_2 = [x_{22}, \cdots, x_{2n}]^T \in \mathbb{R}^{n-1}$.

The sliding surface $s \in \mathbb{R}^{n-1}$ is designed as

$$s = e_1 + \alpha e_2 \tag{33}$$

where $\alpha \in \mathbb{R}$ is a positive constant.

Assumption 4. *The control input u_1 of the leader is bounded such that $|\dot{x}_{21}| \leq \bar{\dot{x}}_{21}$, where $\bar{\dot{x}}_{21} \in \mathbb{R}$ is a known positive constant.*

Theorem 2. *For n-agents connected with a directed graph that has a directed spanning tree, there exists a controller for the leader-follower system in (28) and the sliding surface defined in (33) such that consensus tracking can be achieved exponentially even if the sign of the control direction, i.e., $sign(g_i)$, is unknown.*

Proof. Consider the Lyapunov candidate function as

$$V = \frac{1}{2}\tilde{s}^T\tilde{s} \tag{34}$$

where the augmented sliding surface is defined as

$$\tilde{s} \triangleq s + \lambda \int_0^t sign\left(s\left(\tau\right)\right) d\tau. \tag{35}$$

Case 1: Consider the case when the sign of g_i is unknown but identical, and the magnitude of g_i is identical for all the agents, i.e., $g_i = g_j \triangleq g \; \forall i, j \neq 1$.

Taking time derivative of the Lyapunov function and using (32), (33), and (35),

$$\dot{V} = \tilde{s}^T\Big(\dot{e}_1 + \alpha\Big(\left(\bar{L} + \bar{B}\right) gU - \bar{B}\mathbf{1}\dot{x}_{21} + \left(\bar{L} + \bar{B}\right)\delta\Big) + \lambda sign\left(s\right)\Big) \tag{36}$$

where $U(t)$ and $\delta(t)$ are defined in (24). From (36), $U(t)$ can be designed as

$$U = \left(\bar{L} + \bar{B}\right)^{-1} \frac{M_c}{|g|} sign\left(\sin\left(\frac{\pi}{\epsilon}\tilde{s}\right)\right) \tag{37}$$

where $M_c = \text{diag}\{M_{c2}, \cdots, M_{cn}\} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix chosen such that each element of vector $M_c\mathbf{1}$ is greater than the corresponding element in $\left|\bar{B}\mathbf{1}\bar{\dot{x}}_{21}\right| + \left|\frac{\lambda}{\alpha}\mathbf{1}\right| + \left|\left(\bar{L} + \bar{B}\right)\bar{\Delta}\right| + |\frac{e_2}{\alpha}|$. The controller in (37) guarantees finite time convergence to the surface $\tilde{s}(t)$. If $sign(U) = -sign(M_c e)$, then $\dot{V}$ is negative definite. From (36) and (37), for a constant vector $\beta \in \mathbb{R}^{n-1}$, the surface $\tilde{s}(t)$ reaches β in finite time and sliding mode occurs on β. Then, it follows from (35) that $s(t)$ goes to zero in finite time. From (33), when $s(t)$ goes to zero, the error

$$\dot{e}_1 = -\frac{1}{\alpha}e_1$$

goes to zero exponentially. Thus, the controller in (37) guarantees consensus tracking among the agents. For higher order system, consensus can be realized in a similar way.

Case 2: The control input gains are not identical and could be time-varying, i.e. $g_i(t) \neq g_j(t) \; \forall i, j \neq 1$ and $i \neq j$.

Taking time derivative of Lyapunov function in (36),

$$\dot{V} = \tilde{s}^T\Big(\dot{e}_1 + \alpha\Big(\left(\bar{L} + \bar{B}\right) GU - \bar{B}\mathbf{1}\dot{x}_{21} + \left(\bar{L} + \bar{B}\right)\delta\Big) + \lambda sign\left(s\right)\Big) \tag{38}$$

where $G = diag\{g_2, \cdots, g_n\} \in \mathbb{R}^{(n-1)\times(n-1)}$. Based on (38), the control input can be designed as

$$U = |G|^{-1}\left(\bar{L} + \bar{B}\right)^{-1} M_d sign\left(\sin\left(\frac{\pi}{\epsilon}\tilde{s}\right)\right) \tag{39}$$

where $M_d = diag\{M_{d2}, \cdots, M_{dn}\} \in \mathbb{R}^{(n-1)\times(n-1)}$ is a positive diagonal matrix chosen such that each element of vector $M_d\mathbf{1}$ is greater than the corresponding element in $\left|\bar{B}\mathbf{1}\bar{\dot{x}}_{21}\right| + \left|\frac{\lambda}{\alpha}\mathbf{1}\right| + \left|(\bar{L} + \bar{B})\bar{\Delta}\right| + \left|\frac{e_2}{\alpha}\right|$.

The controllers in (37) and (39) guarantee exponential consensus tracking among agents with second order dynamics. ☐

V. Simulation Results

A simulation is performed with four agents $i = \{1, 2, 3, 4\}$, where (1) is the leader and $(2, 3, 4)$ are the followers. The Laplacian matrix of the followers and the interconnections among the leader and the followers can be expressed using the adjacency matrix as

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad \bar{L} = \begin{bmatrix} 2 & -1 & -1 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}.$$

The diagonal matrix $\bar{B}$ serving as the interconnection between the leader and the followers is given by

$$\bar{B} = diag\left\{0, 0, 1\right\}$$

The control inputs (25) and (37) for the first order and second order system, respectively, are used in the simulation results.

A. *Consensus of the First Order Systems*

The velocity of the leader is given as

$$\dot{x}_1 = u_1 \quad u_1 = \cos(t/10)\sin(t)$$

and the kinematics of the followers are described in equation (14). The initial conditions are

$$x\left(0\right) = \left[\begin{array}{cccc} -0.796 & 0.221 & -0.922 & -0.981 \end{array}\right]^T.$$

The parameters chosen for the control input in (25) are given as following diagonal matrices:

$$\lambda = diag\{0.2, 0.12, 0.12\} \qquad M_a = diag\{3, 3, 3\}$$
$$\varepsilon = diag\left\{\frac{10}{3}, 5, 4\right\} \qquad G = diag\{1, -1, 1\}$$

and $\delta_i(t)$ is considered to be a zero mean Gaussian noise of standard deviation 0.5.

Fig. 2 shows the trajectories of the leader and the followers. It can be seen that consensus is reached at around $t = 15$s. Fig. 3 shows the distance between the leader and followers, and it can be seen from Fig. 3 that the distance goes to zero when consensus is reached.

B. *Consensus of the Second Order Systems*

For the second order system, the leader dynamics are chosen to follow a sinusoidal trajectory

$$\dot{x}_{11} = x_{12} \quad \dot{x}_{12} = u_1 \quad u_1 = \cos(t/2).$$

The initial conditions are

$$x_1 = [0.533 \; -0.851 \; -0.664 \; -0.009]^T$$
$$x_2 = [0\,0\,0\,0]^T.$$

The parameters chosen for the control input in (37) are as follows:

$$\lambda = diag\{0.2, 0.12, 0.12\} \qquad M_c = diag\{3, 3, 3\}$$
$$\varepsilon = diag\left\{\frac{10}{3}, 5, 4\right\} \qquad G = diag\{-1, 1, -1\}$$
$$\alpha = 1 \qquad \bar{\Delta}_i = 1.1$$

and $\delta_i(t)$ is considered to be a zero mean Gaussian noise of standard deviation 1.

Fig. 4 shows the trajectories of the leader and the followers. It can be seen that consensus is reached at around $t = 10$s. Fig. 5 shows the distance between the leader and followers.

VI. Conclusions

In this paper, a novel sliding surface is presented for leader-follower multi-agent systems with sign uncertainty in the control input matrix and non-vanishing disturbances. The sliding surface ensures finite time convergence for single integrator multi-agent systems and exponential convergence for second order multi-agent systems.

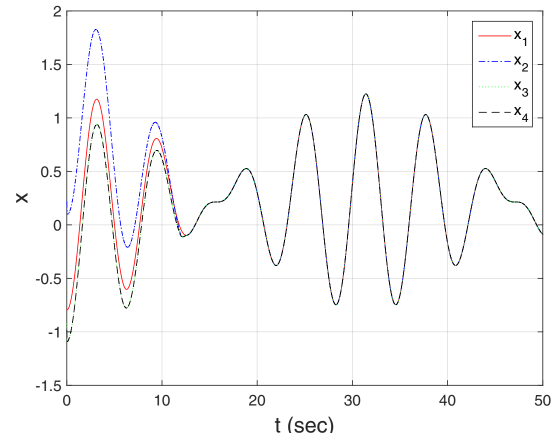

Fig. 2: Trajectories of the followers and leader for the first order system.

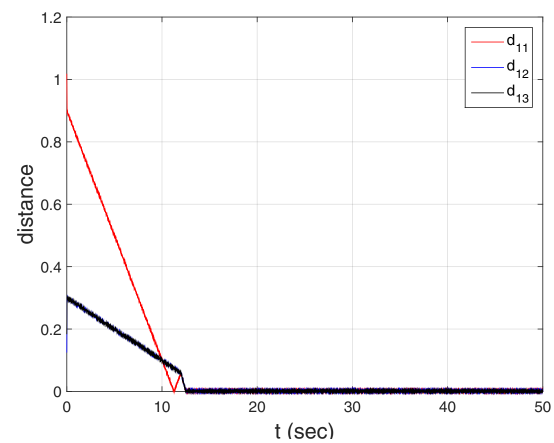

Fig. 3: Distance between the followers and leader for the first order system.

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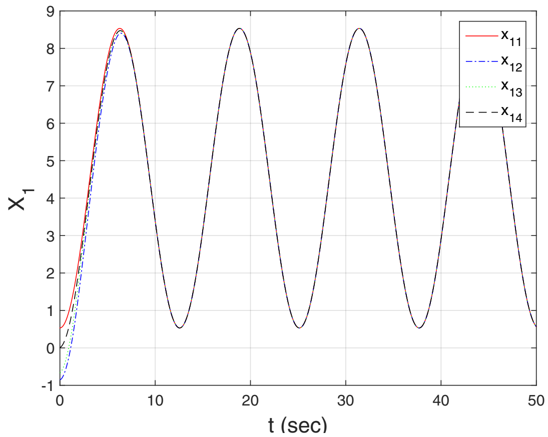

Fig. 4: Trajectories of the followers and leader for the second order system.

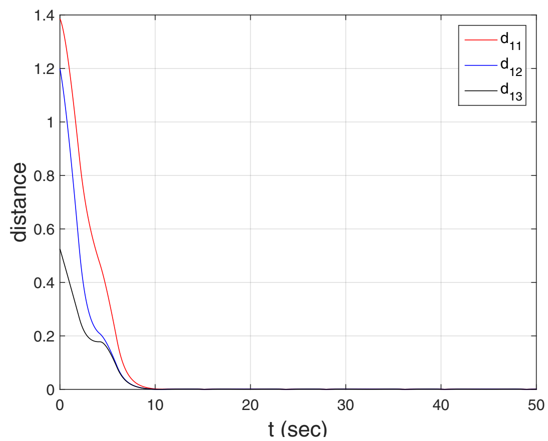

Fig. 5: Distance between the followers and leader for the second order system.

tation," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 42, no. 2, pp. 444–457, April 2012.

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